

Answer the following questions. Calculators & mobile phones are not allowed.

Evaluate the following integrals:

(i)  $\int \frac{\ln(x+1)}{\sqrt{x+1}} dx$  (4 points)

(ii)  $\int \frac{x^{\frac{3}{2}} dx}{x+1}$  (4 points)

(iii)  $\int \frac{dx}{\sin x - \cos x - 1}$  (4 points)

(iv)  $\int \frac{(x-1)}{(x^2-2x-3)^{\frac{3}{2}}} dx$  (5 points)

(v)  $\int \frac{5x^2 + x + 6}{(x+1)(x^2+4)} dx$  (5 points)

Determine whether the integral converges or diverges, and if it converges, find its value.

(i)  $\int_0^{\infty} \frac{dx}{e^x + e^{-x}}$  (4 points)

(ii)  $\int_0^{\frac{\pi}{4}} (\sin x)^{-\frac{1}{2}} (\cos x)^{-\frac{1}{2}} dx$  (4 points)

Find  $\lim_{x \rightarrow \infty} (1 - e^{-x})e^x$  (5 points)

Given that  $f$  is a continuous function and  $f(0) = e$ , find

$\lim_{x \rightarrow 0} \left( \frac{1}{x} \int_0^x f(t) dt \right)$  (5 points)

Good Luck